Solutions to JEE MAIN – 5 |JEE - 2024

PHYSICS

SECTION-1

1.(B) Take coordinates as : A (0, 0) B (a, 0), C (a, a) D (0, a)

Coordinate of COM : $\left(\dfrac{2(0)+1(a)+2(a)+1(0)}{6}, \dfrac{2(0)+1(0)+2(a)+1(a)}{6}\right)$

COM : $\left(\dfrac{a}{2}, \dfrac{a}{2}\right) \Rightarrow$ Distance $= \dfrac{a}{\sqrt{2}}$

2.(D) $I_{\text{requred}} = I_{\text{diagonal}} + M\left(\dfrac{\text{diagonal length}}{2}\right)^2$

$$= \frac{ML^2}{12} + M\left(\frac{L}{\sqrt{2}}\right)^2 = \frac{7ML^2}{12}$$

3.(A) $m_A gx - \dfrac{1}{2}kx^2 = \Delta k = 0 \quad \therefore \quad x = \dfrac{2m_A g}{k}$

Also, $m_B g = kx$ or $x = \dfrac{m_B g}{k}$

$\therefore \quad 2m_A = m_B$ or $\dfrac{m_A}{m_B} = \dfrac{1}{2}$

4.(A) $I_{\text{required}} = I_{\text{plate}} + I_{\text{Disc}}$

$$= \frac{M(2a)^2}{3} + \left(\frac{Ma^2}{4} + Ma^2\right) \qquad = \frac{31}{12}Ma^2$$

5.(B) $\dfrac{dU}{dx} = 4x^3 - 4x = 0 \quad \Rightarrow \quad x = 0, \ \pm 1$

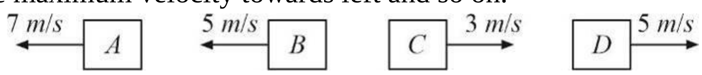
$$\frac{d^2U}{dx^2} = 12x^2 - 4 \qquad \therefore \qquad \left.\frac{d^2U}{dx^2}\right|_{x=0} = -4 \text{ (unstable equation)}$$

6.(B) In case of a light rod, m is constrained to remain in the vertical circle.
At highest point, speed should be zero
Applying work energy then:

$$W_{mg} = \Delta k$$

$$-2mgl = 0 - \frac{1}{2}mv^2 \qquad \therefore \qquad v = \sqrt{4gl}$$

7.(B) If collision is elastic and colliding masses are identical then their velocities get interchanged.
Hence, the right most block (block D) will have maximum velocity toward right. The left most block (block A) will have maximum velocity towards left and so on.



8.(C) Linear momentum is conserved in all types of collisions but kinetic energy is not conserved in all type of collisions. Kinetic energy is conserved in elastic collisions but not conserved in inelastic collisions.

9.(A) $d_1 = u_x t_1$ (Horizontal distance travelled by the ball while reaching the wall)

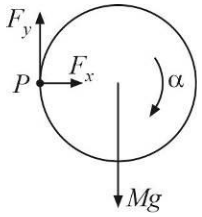
$d_2 = eu_x t_2$ (Horizontal distance travelled by the ball while reaching the ground from wall)

$\because \quad d_1 = d_2 \quad \therefore \quad u_x t_1 = eu_x t_2 \quad \therefore \quad e = \frac{t_1}{t_2}$

10.(C) $\tau_P = I_P \alpha$

or $MgR = \frac{3}{2}MR^2\alpha$

or $\alpha = \frac{2g}{3R}$ (Statement I)



$a_{CM,t} = \alpha R = \frac{2g}{3}$

$F_t = Mg - F_y = Ma_{CM,t} \Rightarrow F_y = Mg - \frac{2Mg}{3} = \frac{Mg}{3}$

$F_c = F_x = Ma_{CM,c} = M\omega^2 r_{CM} = 0$ (Since $\omega = 0$)

$\Rightarrow \quad F_x = 0 \quad \therefore \quad F_{\text{hinge}} = F_y = \frac{Mg}{3}$ (Statement II)

11.(D) Surface is smooth. Therefore, there is no impulsive force along the surface to change horizontal component of ball's velocity. The ball will have non-zero horizontal velocity after collision.

12.(B) $\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = \frac{1}{2}kx^2$ or $mv^2 = \frac{1}{2}kx^2$ or $kx^2 = 2mv^2$ or $x = \sqrt{\frac{2mv^2}{k}}$

13.(A) $\theta = 30°$ is less than angle of repose $(\theta_{max} = \tan^{-1}(\mu) = 37°)$

Hence, block will not slip

Hence friction will be static

$\therefore \quad f_s = mg\sin\theta$

$\therefore \quad \tau$ due to f_s about centre $= f_s \cdot \frac{a}{2} \quad = mg\sin\theta \cdot \frac{a}{2}$

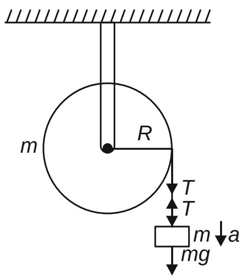
14.(B) Let platform has moved by x distance towards right

$\therefore \quad m_A \times x_A = m_B \times x_B$

or $40 \times \left(\frac{L}{2} + x\right) = 60 \times \left(\frac{L}{2} - x\right)$ or $\frac{4L}{2} + 4x = \frac{6L}{2} - 6x$

or $10x = \frac{2L}{2} = L$ or $x = \frac{L}{10}$

15.(B) For the mass m, $mg - T = ma$



As we know, $a = R\alpha$

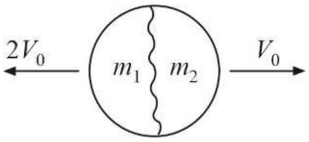
So, $mg - T = mR\alpha$ … (i)

As torque $= T \times R = mR^2\alpha$ … (ii)

From Eqs. (i) and (ii), we get $a = g/2$

Hence, the body will fall with an acceleration $\frac{g}{2}$.

16.(C) $m_1 = 200\ g$ and $m_2 = 400\ g$



Energy released during explosion $= \Delta K_{\text{Gain}} = \frac{1}{2}\mu V_r^2$

or $\quad 6\times10^3 = \frac{1}{2}\times\frac{0.2\times0.4}{0.6}\times(3V_0)^2$

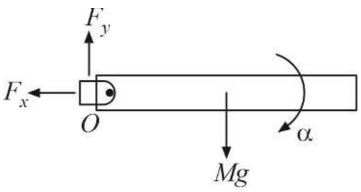
$\Rightarrow \quad V_0^2 = 10^4 \quad \Rightarrow \quad V_0 = 100\ m/s$

17.(D) Stationary object must be in rotational equilibrium

$\therefore \quad \vec{\tau}_{net,\,O} = 0$

Torque due to tension forces about *O* is zero
Hence, torque due to weight about *O* is zero.

18.(C) $\tau_0 = I_0\alpha$ $\quad$ or $\quad mg\frac{L}{2} = \frac{ML^2}{3}\cdot\alpha$



or $\quad \alpha = \frac{3g}{2L}$ $\quad$ or $\quad a_{CM,t} = \frac{\alpha L}{2} = \frac{3g}{4}$

$F_t = mg - F_y = Ma_{CM,t} = \frac{3mg}{4} \quad \therefore \quad F_y = \frac{mg}{4}$

$F_c = F_x = Ma_{CM,\,c} = M\omega^2 r_{CM} = M\cdot\frac{g}{2L}\cdot\frac{L}{2} = \frac{mg}{4}$

$\therefore \quad F_{\text{hinge}} = \sqrt{F_x^2 + F_y^2} = \frac{mg}{2\sqrt{2}}$

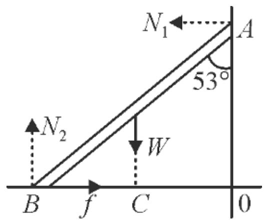
19.(C) The forces acting on the ladder are shown in figure. They are

(A) its weight *W*,

(B) normal force N_1 by the vertical wall,

(C) normal force N_2 by the floor and

(D) frictional force *f* by the floor.



Taking horizontal and vertical components,

$N_1 = f \quad \ldots$ (i)

and $\quad N_2 = W. \quad \ldots$ (ii)

Taking torque about *B*, $\quad N_1(AO) = W(CB)$

or, $\quad N_1(AB)\cos 53° = W\frac{AB}{2}\sin 53°$ $\quad$ or, $\quad N_1\frac{3}{5} = \frac{W}{2}\frac{4}{5}$

or, $\quad N_1 = \frac{2}{3}W = \frac{2}{3}\times100 = \frac{200}{3}N$

20.(C) When the block is released, the spring pushes it towards right. The velocity of the block increases till the spring acquires its natural length. Thereafter, it moves with constant velocity constant velocity.

Initially, the compression of the spring is $L_0/2.$ When the distance of the block from the wall becomes *x*, where $x < L_0,$ the compression is $(L_0 - x).$ Using the principle of conservation of energy,

$\frac{1}{2}k\left(\frac{L_0}{2}\right)^2 = \frac{1}{2}k(L_0-x)^2 + \frac{1}{2}mv^2.$ $\quad$ Solving this, $v = \sqrt{\frac{k}{m}}\left[\frac{L_0^2}{4} - (L_0-x)^2\right]^{1/2}.$

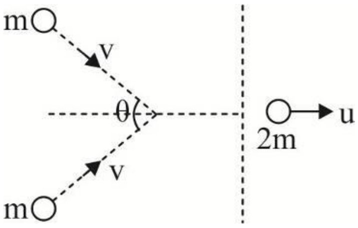
SECTION-2

1.(10) Thrust force, $F_{th} = \frac{dm}{dt} V_{\text{relative}}$

$= 0.5 \times \sqrt{12^2 + 16^2}$

$= 0.5 \times 20 \qquad = 10\ N$

2.(120) $2mV\cos\frac{\theta}{2} =$ net initial momentum

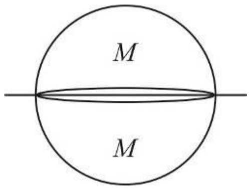


$\because$ no external force act $\left[2mv\cos\frac{\theta}{2} = 2mu\right]$

via conservation of linear momentum

$\Rightarrow \quad u = V\cos\frac{\theta}{2}$; Given $u = \frac{V}{2} \Rightarrow \theta = 120°$

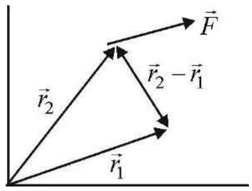
3.(7) $2I_{\text{required}} = \frac{2}{5}\cdot(2M)R^2$



$\therefore \quad I_{\text{requried}} = \frac{2}{5}MR^2$

$\therefore \quad \alpha + \beta = 7$

4.(195) $\vec{\tau} = (\vec{r}_2 - \vec{r}_1) \times \vec{F}$; Torque about point

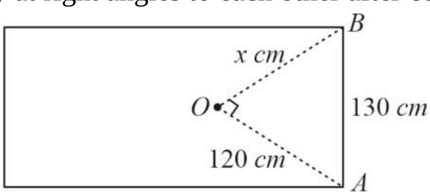


$\vec{\tau} = (\vec{r}_2 - \vec{r}_1) \times \vec{F}$; $\qquad \vec{r}_2 = 4\hat{i} + 3\hat{j} - \hat{k}$

$\vec{r}_1 = \hat{i} + 2\hat{j} + \hat{k}$; $\qquad \vec{\tau} = (3\hat{i} + \hat{j} - 2\hat{k}) \times (\hat{i} + 2\hat{j} + 3\hat{k})$

$\vec{\tau} = 7\hat{i} - 11\hat{j} + 5\hat{k} \ = 10\mu s$; $\qquad \sqrt{x} = \sqrt{195} \Rightarrow x = 195$

5.(50) When two identical bodies collide obliquely and elastically with one of the bodies being at rest before collision, then they move away at right angles to each other after collision.



$x^2 = 130^2 - 120^2 \qquad$ or $\qquad x = 5$

CHEMISTRY

SECTION-1

1.(B) CO and B_2H_6 have same molecular mass. The diffusion of lighter gas will be more. Thus order of rate of diffusion will be: $H_2 > CH_4 > CO = B_2H_6$

2.(A) $PV = \frac{w}{M}RT$ or $P = \frac{w}{VM} \cdot RT$ or $P = \frac{d}{M}RT \Rightarrow P \alpha d$

At sea level atmospheric pressure is high thus density is high (volume decreases).

3.(B) $z = \frac{PV}{nRT} = \frac{PVM}{wRT} = \frac{P \times M}{\rho RT}$

$= \frac{2 \times 10^5 \times 18 \times 10^{-3}}{0.6 \times 8.314 \times 373} = 1.94$

4.(B) $PV = K$

$\log P = \log K + \log \frac{1}{V}$

Also, $PV = RT$

$\Rightarrow \quad RT = 0.0821 \times 400 \quad \Rightarrow \quad K = 32.84$

5.(A) $\Delta H = H_P - H_R$

For Exothermic reaction ΔH is negative thus $H_P < H_R$

6.(B) The increase in temperature flattens the peak.

7.(B) The amount of heat either evolved or absorbed when one gram mole of a substance is formed from its constituent elements in their reference state at 25°C and 1 atm pressure is known as the standard heat of formations (ΔH_f°).

8.(A) $\Delta H = \Delta U + \Delta PV \quad (\because \Delta U = \Delta E)$

$\Delta U = 30.0$ L-atm

$\therefore \quad \Delta H = 30 + (20 - 6) = 44$ L-atm

$= 44 \times 101.2 \, (1 \text{ L-atm} = 101.2 \text{ J}) = 4.45 \text{ kJ}$

9.(A) Heat evolved in first process is twice to second

$Q_1 = 2Q_2$

$1T_1 = 2(0.5T_2) \quad \Rightarrow \quad T_1 = T_2$

10.(B) Lower is heat of neutralization more is dissociation energy, weaker is acid.

11.(C) $n_1 = n_2$

$\frac{1}{32} = \frac{2.375}{M_2}$

$M_2 = 76$

Mw for $CS_2 = 76$

12.(A) Average K.E. $= \frac{3 \times n \times R \times T}{2 \times \text{Av. no}} = \frac{3 \times 8.314 \times 10^7 \times 300}{2 \times 6.023 \times 10^{23}} = 6.21 \times 10^{-14}$ erg

13.(C) $u_{rms} = \sqrt{\frac{3RT}{M}} \quad \therefore \quad 3 \times \sqrt{R} = \sqrt{\frac{3RT}{M}}$ or $9R = \frac{3RT}{M}$ or $M = \frac{3 \times 300}{9} = 100 \text{ kg mol}^{-1}$

14.(A) $\Delta H = 6 \times 410 + 700 \times 3 - 6 \times 410 - 6 \times 360$

$= -60$ for 3 moles

$\therefore$ For 1 mole = –20 kJ/mole

15.(C) Heat evolved during 1 gram decomposition of NH_4NO_3

$= ms\Delta T$

$= 1 \times 1.23 \times 6.32$

$\therefore$ Heat evolved during 1 mole decomposition

$= 1 \times 1.23 \times 6.32 \times 80$

$= 621.89 \text{ kJ mol}^{-1}$

16.(A) $P(V-b) = RT$

$\therefore \quad P = \dfrac{R \times T}{(V-b)}$, Slope $= \dfrac{R}{(V-b)}$

$PV - Pb = RT$

$V = \dfrac{RT}{P} + b$, Slope = R/P and intercept b

$Z = \dfrac{PV}{RT} = 1 + \dfrac{Pb}{RT}$

Z > 1, i.e., repulsive forces predominates

17.(C) $-1941 = -1273 + 3(-241.8) - (\Delta_f H)_{B_2H_6}$

$(\Delta_f H)_{B_2H_6} = -1273 - 725.4 + 1941 = -57.4$

18.(D) $2CO_{(g)} + O_{2(g)} \rightarrow 2CO_{2(g)}$; $\Delta H_1 = -P \text{ kJ}$

$C_{(s)} + O_{2(g)} \rightarrow CO_{2(g)}$; $\Delta H_2 = -Q \text{ kJ}$

$C_{(s)} + \dfrac{1}{2}O_{2(g)} \rightarrow CO_{(g)}$; $\Delta H_3 = ?$

$\Delta H_3 = \Delta H_2 - \dfrac{1}{2}\Delta H_1$

$= -Q - \left(-\dfrac{P}{2}\right) = -Q + \dfrac{P}{2} = \dfrac{P-2Q}{2}$

19.(A) F^- shows extensive hydration

$\therefore \quad \Delta H_{neutralisation} > -13.7$,

For CH_3COOH, $\Delta H_{neutralisation} < -13.7$

20.(B) 1 Meq. of NaOH = 20 ; Meq of $H_2SO_4 = 20$

II Meq. of NaOH = 10 ; Meq. of $H_2SO_4 = 10$

In I : $\Delta H = -\dfrac{13.7 \times 10^3 \times 20}{1000}$ = 274 cal.

In II : $\Delta H = -\dfrac{13.7 \times 10^3 \times 10}{1000}$ = 137 cal.

SECTION-2

1.(53.68)

Bond enthalpy of C – C bond

= Enthalpy required to break C_2H_6 into gaseous atoms –6 (bond enthalpy of C – H bond)

$= 2839.2 \text{ kJ mol}^{-1} - 6\ (410.84 \text{ kJ mol}^{-1})$

$= 373.98 \text{ kJ mol}^{-1}$

Bond enthalpy of C = C bond = Enthalpy required to break C_2H_4 into gaseous atoms –4 (bond enthalpy of C – H)

$$= 2275.2 \text{ kJ mol}^{-1} - 4\ (410.87 \text{ kJ mol}^{-1})$$

$$= 631.72 \text{ kJ mol}^{-1}$$

For the formation of benzene having Kekule structure, we have to form 3 (C – C) bond, 3 (C = C) bonds and 6 (C – H) bonds for which enthalpy released is

$[3\ (-373.98) + 3(-631.72) + 6(-410.87)] = -5482.32 \text{ kJ mol}^{-1}$

But the given value of

$= \Delta_f H^\circ \text{ (actual)} - \Delta_f H^\circ \text{ (Kekule structure)}$

$= (-5536 + 5482.32) = -53.68 \text{ kJ mol}^{-1}$

2.(4) Given $V = 0.02\ m^3,\ T = 300\ K,\ P = 1\times10^5\ Nm^{-2},\ R = 8.314\ J$

Let a and b g be mass of Ne and Ar respectively

Thus a + b = 28 … (i)

Also total mole of Ne and $Ar = \frac{a}{20} + \frac{b}{40}$

Thus from PV = nRT

$$\therefore \quad 1\times10^5 \times 0.02 = \left[\frac{a}{20} + \frac{b}{40}\right] \times 8.314 \times 300$$

$$2a + b = 32.0 \quad \ldots \text{(ii)}$$

By equations (i) and (ii)

$a = 4g\ ;\ b = 24g$

$a + b = 28$

3.(181.29)

Amount of water present on the body of swimmer $= 80g = \frac{80}{18}$ mole

Thus, heat required to evaporate $\frac{80}{18}$ mole

$H_2O = 40.79 \times \frac{80}{18} = 181.288\,kJ = 181.29\,kJ$

4.(125.07)

We have to find ΔH for

$$NH_4NO_{3(s)} \longrightarrow N_2O_{(g)} + 2H_2O_{(l)}\ ; \quad \Delta H^\circ = ?$$

$$\Delta H^\circ_{reaction} = \Delta H^\circ_{product} - \Delta H^\circ_{reactants}$$

$$= \Delta H^\circ_{N_2O} + \Delta H^\circ_{H_2O} \times 2 - \Delta H^\circ_{NH_4NO_3}$$

Given, $\Delta H^\circ_{N_2O} = +81.46\,kJ,\ \Delta H^\circ_{H_2O} = -285.8\,kJ,\ \Delta H^\circ_{NH_4NO_3} = -367.54\,kJ$

$$\therefore \quad \Delta H^\circ_{reaction} = +81.46 + 2(-285.8) - (-367.54)$$

$$\Delta H^\circ = -122.6\,kJ$$

Further $\Delta H^\circ = \Delta E^\circ + \Delta n_g RT \quad (\Delta n_g = 1 - 0 = 1, R = 8.314\ J\ K^{-1}\ mol^{-1},\ T = 298\,K)$

$$\therefore \quad -122.6\times10^3 = \Delta E + 1\times8.314\times298$$

$$\therefore \quad \Delta E^\circ = -125077\ J = 125.077\ kJ = 125.08\,kJ$$

5.(55.7)

On the basis of definition of standard heat of combustion, following thermochemical equation is written for combustion of $C_3H_8(g)$

$C_3H_8(g) + 5O_2(g) \rightarrow 3CO_2(g) + 4H_2O(l), \qquad \Delta H = ?\ldots(1)$

From given data (for combustion of $H_2(g)$ and graphite)

(i) $H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(g),\ \Delta H = -285.8\ \text{kJ/mol}$

It is also known as ΔH_f of H_2O

(ii) $C_{(graphite)} + O_2(g) = CO_2(g), \quad \Delta H = -393.5\ \text{kJ / mol}$

It is also known as ΔH_f of $CO_2(g)$

For ΔH of thermochemical equation number (i)

$\Delta H = [(3 \times \Delta H_f \text{ of } CO_2(g) + 4 \times \Delta H_f \text{ of } H_2O(l))] - [\Delta H_f \text{ of } C_3H_8(g) + 5 \times \Delta H_f \text{ of } O_2(g)]$

$= [(3 \times -393.5 + 4 \times -285.8) - (-103.8 + 5 \times 0)]$

$= -2219.9\ \text{kJ/mol}$

For following thermochemical equation:

$C_3H_8(g) + H_2(g) \longrightarrow C_2H_6(g) + CH_4(g),\ \Delta H = ?$

On the basis of standard heat of combustion,

$\Delta H = -[(\Delta H \text{ combustioin of } C_2H_6 + \Delta H \text{ of combustion of } CH_4)$

$-(\Delta H \text{ of combustion of } C_3H_8 + \Delta H \text{ of combustion of } H_2)]$

$= -[(-1560.0) + (-890.0) - (2219.9) + (-285.8)]$

$= -[-2450.0 + 2505.7]\ \text{kJ/mol}$

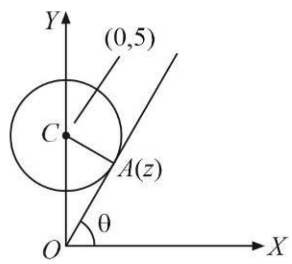
$= -55.7\ \text{kJ mol}^{-1}$

MATHEMATICS

SECTION-1

1.(A) We have, $|z-5i|\le 1$

Let $\theta=\angle AOX=\min.\text{amp}(z)$



Therefore, $\angle AOC=90°-\theta$

$\Rightarrow \quad \sin(90°-\theta)=\dfrac{1}{5}$

$\Rightarrow \quad \cos\theta=\dfrac{1}{5}$

Therefore, $z=OA\cos\theta+iOA\sin\theta$

$\Rightarrow \quad z=\sqrt{5^2-1}\left(\dfrac{1}{5}\right)+i\sqrt{5^2-1}\sqrt{1-\dfrac{1}{5^2}}$

$=\dfrac{2\sqrt{6}}{5}(1+i2\sqrt{6})$

2.(B) Complex equation $\dfrac{(1+i)x-2i}{3+i}+\dfrac{(2-3i)y+1}{3-i}=i$

The given equation may be written as follows:

$\dfrac{(3-i)(1+i)x-(3-i)2i+(3+i)(2-3i)y+(3+i)}{(3+i)(3-i)}=i$

or $\quad (4+2i)x+(9-7i)y-5i+1=i(3^2-i^2)=10i$

or $\quad (2x-7y-5)i+4x+9y+1=10i$

Equation real and imaginary parts, we get

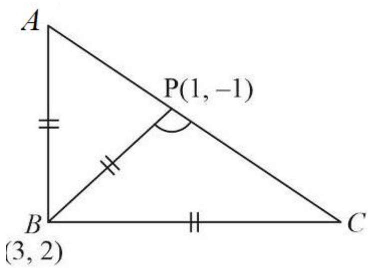
$2x-7y-5=10$

or $\quad 2x-7y=15 \quad \ldots$ (i)

and $\quad 4x+9y=-1 \quad \ldots$ (ii)

Solving equations (i) and (ii) $y=-\dfrac{31}{23}$

3.(B)



$\dfrac{PB}{PA}=e^{i\frac{\pi}{2}}=i$ and $\dfrac{PC}{PB}=e^{i\frac{\pi}{2}}=i$

$\dfrac{PA}{PB}=-i$

$\dfrac{Z_A-(1-i)}{2+3i}=-i$

$Z_A-(1-i)=-2i+3$

$\dfrac{Z_C-(1-i)}{2+3i}=i$

$Z_C=(1-i)+2i-3$

$Z_C=-2+i$

$Z_A = 4-3i$

4.(B) $\left|z^2-1\right| = |z|^2+1$

$\Rightarrow$ Let $z = x+iy$

$\left|(x+iy)^2-1\right| = |x+iy|^2+1$

$\left|(x^2-y^2-1)+(2xy)i\right| = (\sqrt{x^2+y^2})^2+1$

$\sqrt{(x^2-y^2-1)^2+4x^2y^2} = x^2+y^2+1$

$(x^2-y^2-1)^2+4x^2y^2 = (x^2+y^2+1)^2$

Solving it

$x = 0$

z lies on the imaginary axis.

5.(D) The given equation is

$$z^2+z+1=0$$

$\Rightarrow$ $z = \omega, \omega^2$

Now, $\left(z+\frac{1}{z}\right)^2+\left(z^2+\frac{1}{z^2}\right)^2+\left(z^3+\frac{1}{z^3}\right)^2+\left(z^4+\frac{1}{z^4}\right)^2+\left(z^5+\frac{1}{z^5}\right)^2+\left(z^6+\frac{1}{z^6}\right)^2$

$= (-1)^2+(-1)^2+(1+1)^2+(-1)^2+(-1)^2+(1+1)^2$

When, we put either $z = \omega^2$, we get the same result as follows:

$1+1+4+1+1+4=12$

6.(D) Let the four points be represented by z_1, z_2, z_3 and z_4 be *A*, *B*, *C*, and *D*, respectively. Since *ABCD* is a square, the midpoint of *AC* = midpoint of *BD*.

$\Rightarrow$ $\frac{1}{2}(z_1+z_3) = \frac{1}{2}(z_2+z_4)$ or $z_1+z_3 = z_2+z_4$

Also, $AB = BC = CD = DA$

$\Rightarrow$ $|z_1-z_2| = |z_2-z_3| = |z_3-z_4| = |z_4-z_1|$

Since diagonals of the square *ABCD* are equal $AC = BD$

or $|z_1-z_3| = |z_2-z_4|$

7.(C) The three lines are concurrent if $\begin{vmatrix} 1 & 2 & -9 \\ 3 & -5 & -5 \\ a & b & -1 \end{vmatrix} = 0$

or $5a+2b=1$

Which implies that the line $5x+2y=1$ passes through (a, b)

8.(C) As the third vertex lies on the line $y = x+3$, its coordinates are of the form $(x, x+3)$

The area of the triangle with vertices (2, 1), (3, –2) and $(x, x+3)$ is given by

$$\frac{1}{2}\left\|\begin{matrix} x & x+3 & 1 \\ 2 & 1 & 1 \\ 3 & -2 & 1 \end{matrix}\right\| = |2x-2| = 5 \qquad \text{(Given)}$$

$\therefore$ $2x-2 = \pm 5$ $\Rightarrow$ $x = \frac{-3}{2}, \frac{7}{2}$

Thus, the coordinates of the third vertex are $\left(\frac{7}{2},\frac{13}{2}\right)$ or $\left(\frac{-3}{2},\frac{3}{2}\right)$

9.(B) $x+2y+4=0$ and $4x+2y-1=0$

$\Rightarrow \quad x+2y+4=0$

and $\quad -4x-2y+1=0$

Here, $\quad (1)(-4)+(2)(-2)=-8<0$

$\therefore$ Bisector of the angle including origin and the acute angle bisector is

$$\frac{x+2y+4}{\sqrt{5}}=\frac{(-4x-2y+1)}{2\sqrt{5}} \quad \Rightarrow \quad 6x+6y+7=0$$

10.(A) $|z_1-3z_2|=|3-z_1\bar{z}_2|$

$\Rightarrow \quad (z_1-3z_2)(\bar{z}_1-3\bar{z}_2)=(3-z_1\bar{z}_2)(3-\bar{z}_1z_2)$

$\Rightarrow \quad |z_1|^2-3z_1\bar{z}_2-3\bar{z}_1z_2+9|z_2|^2$

$=9-|z_1|^2|z_2|^2-3z_1\bar{z}_2-3\bar{z}_1z_2$

$=|z_1|^2+9|z_2|^2+|z_1|^2|z_2|^2=9$

$=(|z_1|^2-9)(|z_2|^2-1)=0$

$=|z_2|=1$

11.(A) $x+iy=\frac{3}{(2+\cos\theta)+i\sin\theta}\times\frac{(2+\cos\theta)-i\sin\theta}{(2+\cos\theta)-i\sin\theta}$

$x+iy=\frac{6+3\cos\theta-3i\sin\theta}{(2+\cos\theta)^2+\sin^2\theta}$

Equating real and imaginary parts

12.(D) $(z^n-1)\equiv(z-1)(z-z_1)(z-z_2)\ldots(z-z_{n-1})$

Taking logarithms and on differentiation

$$\frac{n\cdot z^{n-1}}{z^n-1}=\frac{1}{z-1}+\frac{1}{z-z_1}+\ldots+\frac{1}{z-z_{n-1}}$$

For z = 3

$$\Rightarrow \quad \frac{n\cdot 3^{n-1}}{3^n-1}=\frac{1}{2}+\frac{1}{3-z_1}+\frac{1}{3-z_2}+\ldots+\frac{1}{3-z_{n-1}}$$

$$\Rightarrow \quad \frac{1}{3-z_1}+\frac{1}{3-z_2}+\ldots+\frac{1}{3-z_{n-1}}=\frac{n\cdot 3^{n-1}}{3^n-1}-\frac{1}{2}$$

13.(C) $ac<0$ and $bc<0$

14.(A) Given, equation of line is $\sqrt{3}x+y+2=0$

$\Rightarrow \quad \sqrt{3}x+y=-2 \quad \Rightarrow \quad -\sqrt{3}x-y=2$

On dividing above equation by $\sqrt{(\text{coefficient of } x)^2+(\text{coefficient of } y)^2}$

i.e., $\quad \sqrt{(-3)^2+(-1)^2}=\sqrt{3+1}=\sqrt{4}=2 \quad \Rightarrow \quad \frac{-\sqrt{3}}{2}x-\frac{1}{2}y=\frac{2}{2}$

$\Rightarrow \quad -\cos 30^\circ x-\sin 30^\circ y=1$ (Convert in the form of $x\cos\alpha+y\sin\alpha=p$)

$\Rightarrow \quad \cos(180^\circ+30^\circ)x+\sin(180^\circ+30^\circ)y=1$

($\because$ $\cos\theta$ and $\sin\theta$ both are negative in third quadrant)

$\Rightarrow \quad (\cos 210°)x + (\sin 210°)y = 1$

On comparing with $x\cos\theta + y\sin\theta = p,$ we get ; $\theta = 210°$ and $p = 1$

15.(C) $ax + by = 1$ will be one of the bisectors of the given line. Equation of bisectors of the given lines are:

$$\frac{3x+4y-5}{5} = \pm\left(\frac{5x-12y-10}{13}\right)$$

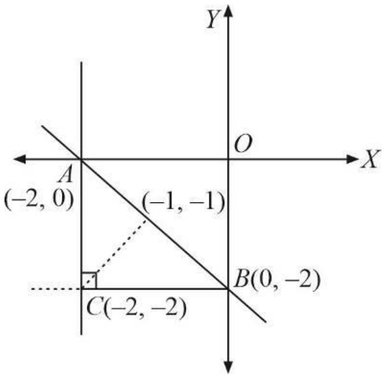
$\Rightarrow \quad 64x - 8y = 115 \qquad$ or $\qquad 14x + 112y = 15$

$\Rightarrow \quad a = \frac{64}{115},\ b = -\frac{8}{115} \qquad$ or $\qquad a = \frac{14}{15},\ b = \frac{12}{115}$

16.(B) $xy + 2x + 2y + 4 = 0$

$\Rightarrow \quad (y+2)(x+2) = 0$

(–1, 1) is equidistant from $(-2, 0),\ (0, -2),\ (-2, -2)$



17.(A) $l_1 : x - 3y = p \quad \Rightarrow \quad m_1 = \frac{1}{3}$; $l_2 = ax + 2y = q$

$\Rightarrow \quad m_2 = -\frac{a}{2}$

$l_3 : ax + y = r \quad \Rightarrow \quad m_3 = -a$

The form a right-angled triangle $m_1 m_2 = -1, m_2 m_3 = -1$ or $m_1 m_2 = -1$

$\Rightarrow \quad \frac{1}{3}\cdot\left(-\frac{a}{2}\right) = -1, \left(-\frac{a}{2}\right)(-a) = -1$

or $\quad \frac{1}{3}(-a) = -1$

$\Rightarrow \quad a = 6,\ a^2 = -2$ (rejected) or $a = 3$

$\therefore \quad (a-6)(a-3) = 0 \qquad \Rightarrow \qquad a^2 - 9a + 18 = 0$

18.(A) O, G, C are collinear and $OG : GC = 1 : 2$. Get G.

$AG : GD = 2 : 1$, where D is the mid point of side opposite to A. Now get D

19.(B) $(x+y)^2 = (x-1)^2 + (y-1)^2$

$2xy = 2 - 2x - 2y$

$x + y + xy = 1$

$x + y + xy + 1 = 2$

$\Rightarrow \quad (x+1)(y+1) = 2$

20.(B) Given equation is $\cos\theta(x + y - 9) + \sin\theta(x - y - 3) = 0$

$\Rightarrow \quad (x + y - 9) + \tan\theta(x - y - 3) = 0,$ which is of the form $L_1 + \lambda L_2 = 0$

Where $L_1 : x + y - 9 = 0$ and $L_2 : x - y - 3 = 0$

Hence the lines will always pass through the point of intersection of $L_1 = 0$ and $L_2 = 0$ i.e., $M(6, 3)$

So its reflection in the line $y = x$ will be (3, 6)

SECTION-2

1.(5) $z^2 = 81 - b^2 + 18bi$

$z^3 = 729 + 243bi - 27b^2 - b^3 i$

$z^2 = z^3 \Rightarrow 243b - b^3 = 18b$ and $243 - b^2 = 18$

$\Rightarrow \quad b = 15$

2.(3) Lines $5x + 3y - 2 + \lambda(3x - y - 4) = 0$ are concurrent at (1, –1) and lines $x - y + 1 + \mu(2x - y - 2) = 0$ are concurrent at (3, 4).

Thus equation of line common to both family is

$$y + 1 = \frac{4+1}{3-1}(x - 1)$$

or $\quad 5x - 2y - 7 = 0$

$\therefore \quad a = 5,\ b = -2 \quad \Rightarrow \quad a + b = 3$

3.(5) $|\alpha|_{\max} = \sqrt{2} + 1,\ |\beta|_{\max} = 6 + \sqrt{13}$

$6|\alpha|_{\max} - |\beta|_{\max} = 6\sqrt{2} - \sqrt{13} = \sqrt{72} - \sqrt{13} = \sqrt{a} - \sqrt{b}$

$\sqrt{b^2 - 2a} = \sqrt{169 - 144} = 5$

4.(6) Since, $x + \frac{1}{x} = 1$

$\Rightarrow \quad x^2 - x + 1 = 0 \quad \Rightarrow \quad x = \frac{1 \pm \sqrt{3}i}{2} \quad \Rightarrow \quad x = -\omega,\ -\omega^2$

Now, $\quad x^{4000} = \omega^{4000}, \omega^{8000} = \omega, \omega^2$

Therefore, $p = x^{4000} + \frac{1}{x^{4000}} = \omega + \frac{1}{\omega}$

or $\quad \omega^2 + \frac{1}{\omega^2}$

$\Rightarrow \quad p = \omega + \omega^2 \quad$ or $\quad \omega^2 + \omega = -1$

Secondly, let $2^{2^n} + 1$ where, $n > 1$

Now, for $n > 1$ we have

$2^n = 4\lambda$, where $k \in N$

Therefore, $q = 2^{4\lambda} + 1 = (16)^{\lambda} + 1$

$\Rightarrow \quad q = (10k + 6) + 1 = 10k + 7$

Therefore, q = the digit at unit's place in $2^{2^n} + 1 = 7$

Hence, $p + q = 6$

5.(3) $3x + 4y = 12 \quad \Rightarrow \quad \frac{x}{4} + \frac{y}{3} = 1$

x-intercept = 4, y-intercept = 3

For new line x-intercept = 8, y-intercept = 3/2

Equation of L is $\frac{x}{8} + \frac{y}{3/2} = 1$, so slope $= -\frac{3}{16}$